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Comparisons of Precession-Nutation Models

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Abstract. We first summarize the key characteristics of the IAU 2000 and IAU 2006 precession-nutation models, (i.e., the IAU 2000 nutation and the P03 precession). Then we discuss some aspects of the numerical ERA-2005 model for Earth-rotation variations in comparison with the semi-analytical IAU models. We point out severe flaws in the ERA model (that have already been noted in a recent comments paper) regarding various aspects of this model, such as the expression for the centrifugal perturbation, Love number formalism, the modeling of the dissipative phenomena, the Earth model, and the estimation of parameters. We finally report on comparisons of those precession-nutation models with VLBI observations and with the INPOP06 numerical integration.

1. Introduction

This paper has been prompted by recent discussions about a comments paper by Mathews, Capitaine and Dehant [1] on the ERA-2005 model [2, 3] that pointed out a number of severe deficiencies in the theory and its fit to data.

The aim of this paper is to make clear the fundamental differences in the main features of the IAU 2000/2006 and the ERA-2005 precession-nutation models and to report on the differences of the models with respect to VLBI observations.

2. Key Characteristics of the IAU Precession-Nutation

2.1. The IAU 2000/2006 Resolutions on Precession-Nutation

The IAU has adopted a high precision model for precession and nutation in two successive steps.

The first step (IAU 2000 Resolution B1.6) was the adoption of the IAU 2000 precession-nutation [4], which has been implemented in the IERS Conventions 2003. This model is composed of a nutation part and a precession part. In addition there are frame bias values between the J2000 mean pole and equinox and the Geocentric Celestial Reference System (GCRS). The IAU 2000A nutation is expected to have an accuracy of about 10 μas for most of its terms. In contrast, the so-called free core nutation (FCN), which is due to geophysical effects and is largely unpredictable, is not part of the model. The precession part, which consists only of corrections to the precession rates of the IAU 1976 precession, was known not to correspond to a dynamical theory; this is why IAU 2000 Resolution B1.6 recommended the development of new expressions for precession consistent with dynamical theories and with IAU 2000A nutation.

The second step (IAU 2006 Resolution B1) was the adoption of the P03 Precession [5, 6] as a replacement to the precession part of the IAU 2000A precession-nutation. The P03 model has provided improved expressions for both the precession of the ecliptic and the precession of the equator, the latter being consistent with dynamical theory, while matching the IAU 2000A precession rate for continuity reasons.

2.2. The Characteristics of the IAU Model

The IAU 2000 nutation is based on the REN2000 solution [7] for the nutation of a rigid Earth model transformed to nutation of a non-rigid Earth model with the MHB2000 "transfer function" [4]. This was derived from the solution of equations obtained by generalization of the SOS equations [8] with Basic Earth Parameters fitted to VLBI data. The semi-analytical series (see IERS Conventions 2003 Tabl. 5.3a and 5.3b) is composed of 1365 luni-solar and planetary terms with "in-phase" and "out-of-phase" components with amplitudes from 17.2 arcsec to 0.1 μas and periods between 3 d and 101 cy.

The IAU 2006 precession [5] was derived from the dynamical equation expressing the motion of the mean pole about the ecliptic pole. The solution is based on values at J2000.0 for the mean obliquity of the ecliptic, ϵ_0, and for the precession rate in longitude and obliquity. The first order contributions to the precession rates have been derived from the MHB values [4] and the P03 value for ϵ_0 (i.e., 84381.406 arcsec). The other contributions to the P03 precession rates are from [9] and [4] and the geodesic precession is from [10]. It takes into account the effect of the Earth's J_2 rate (mostly due to the post-glacial rebound), with a value [9] for dJ_2/dt of $-\ 3.0\ \times 10^{-11}$/y (i.e. $de/dt = -9.1 \times 10^{-11}$/y); this is responsible for an additional contribution of about -7 mas/cy^2 to the t^2 term in the precession in longitude.

The P03 value for the Earth's dynamical flattening, $H_d = 3.27379448 \times 10^{-3}$ is quite consistent with the MHB value of $(3.27379492 \pm 0.00000120) \times 10^{-3}$. The largest differences between the P03 semi-analytical expressions for the precession quantities and expressions corresponding to other recent precession models [11, 12] are of the order of a few mas/cy for the precession rates and 7 mas/cy^2 for the t^2 term in longitude due to the J_2 rate contribution which is taken into account in P03 while it is not in other models (e.g. [11]).

The largest uncertainty in the IAU precession is due to the current uncertainty (of the order of 1 mas/cy) in the observed precession rate. Additionally, the complexity of the J_2 variations causes a large uncertainty in the J_2 rate value and consequently in the quadratic term of the precession in longitude [14]. While the secular decrease in J_2 has to be taken into account to provide a precession model valid over the long term, it cannot be predicted over periods of 10 years with a good accuracy. A recent study [13] of the observed variations of J_2 over the past 28 years based on a large set of satellite laser ranging data showed that in addition to the secular decrease (of about -2.8×10^{-11}/y, i.e. $de/dt \approx -8.5\times10^{-11}$/y), there are seasonal annual variations (with a mean amplitude of 2.9 $\times10^{-10}$), significant inter-annual variations with timescales of 4-6 years (related to strong El Niño-SO events) and a variation with a period of 21 years and an amplitude of about 1.4 $\times10^{-10}$ (with a minimum in Dec. 1988). This latter variation is large enough to mask the secular change in J_2 during certain time spans of VLBI observations.

3. Comparison Between the IAU and ERA Models

3.1. Centrifugal Perturbation and Love Number Formalism

MHB 2000 is based on the SOS equations of Sasao-Okubo-Saito [8] for the variations in rotation of the Earth's mantle and fluid core. The following concepts are fundamental to the SOS/MHB theories:

1) The tidal contribution to the centrifugal potential is only the incremental part arising from the time dependent part dΩ of the angular velocity vector Ω, generated by the action of the perturbing potential; the deformational response to it is characterized by the Love number k_2.

2) The response to forcing by the constant centrifugal potential due to $\Omega_0 = \Omega - d\Omega$ is very far from that of an elastic solid; it is characterized by the fluid/secular Love number $k_f = k_s \approx 0.9 \approx 3k_2$, appropriate to a fluid-like behavior, and is responsible for the Earth's dynamical ellipticity e.

3) It follows from the SOS/MHB equations that the scale factor for the precession rate is $S_{MHB} = H_d = e/(1+e)$.

ERA-2005, in contrast, starts with the "revised SOS equations" based on the following concepts:

1) the centrifugal potential due to the full angular velocity vector $\Omega = \Omega_0 +$ dΩ is treated as a "tidal" perturbation (though Ω_0 is quite unrelated to tidal potentials),

2) the Earth's deformational response to constant forcing is just like its elastic response to periodic forcing at tidal frequencies; the Love number k representing the response has the same value $k_2 \approx 0.3$ in both cases,

3) it follows from the revised SOS equations that the scale factor for the precession rate is $S_{ERA} = e/(1 + e - e\sigma/3)$, with $\sigma = k_2/k_s \approx 0.3$.

The dependence of the scale factor on the deformability (through k_2/k_s) is a radical change, which modifies significantly the value of the precession rate and nutation amplitudes. Using the ERA value S_{ERA} would give a difference with respect to IAU 2000 of -0.14 arcsec/cy in the precession rate and 0.4 mas in the 18.6 y nutation. Note that this is at odds with the difference of (-0.82 ± 0.22) mas/cy between the linear trend in the residuals between the ERA and IAU time series that is reported in the ERA-2005 paper [3] (i.e. ≈ 170 times lower), which raises serious doubts about the physical trustworthiness of the ERA nutation-precession time series.

3.2. Modeling of the Dissipative Phenomena

MHB 2000 spells out the mechanisms which produce the out-of-phase parts of nutation amplitudes and models them on the basis of actual observational data on the various phenomena involved:

1) mantle anelasticity is based on the Wahr and Bergen model, deduced from seismological observations; this model is incorporated into the expressions for the increments of inertia of the whole Earth and the fluid core,

2) ocean tide heights are those from CSR4 tables based on space-geodetic observations,

3) core-mantle and outer core to inner core couplings due to the magnetic fields at the boundaries is modeled from theory and the complex coupling constants K^{CMB} and K^{ICB}.

ERA-2005 in contrast invokes no specific physical mechanisms for the dissipation:

1) it models dissipation in the Earth in terms of three phase delay parameters $\delta, \delta_c, \delta_i$ for the deformational responses in the mantle and the outer and inner fluid cores to tidal perturbations, plus the dissipative part κ_{dis} of a CMB coupling,

2) the ocean tide effects in nutations are introduced through an increment dk_2 to the Love number k_2, which is not constrained by any ocean-tides observations. The expression for that increment is $dk_2 = k_2^{(1)} \cos\theta_B + k_2^{(2)} \cos^2\theta_B$, which depends only on the co-latitude θ_B of the tide producing body in the sky at that instant and has identical values at all points of the globe at any instant. This is very unphysical because the ocean tide is well-known to be highly non-uniform over the Earth's surface.

Note that a similar-looking formula of Kaula [15] has no dependence on the co-latitude of the tide producing body as in ERA-2005; it depends instead on the co-latitude of the observation site, which makes sense physically.

None of the information available from seismological studies and from the detailed observations of ocean tides has been used in the modeling of dissipation. In fact the ERA ocean tide model has no dissipation at all, since $dk_2^{(1)}, dk_2^{(2)}$ are real.

As realistic models for dissipation are essential for computing the out-of-phase nutations, an unphysical model cannot represent these terms correctly.

3.3. Earth Model

MHB 2000 uses spherically symmetric classical models of the radial variation of the properties of a non-rotating Earth, based on seismological data (e.g. PREM). Models of the ellipsoidal structure of the interior of the steadily rotating Earth are derived from such models using hydrostatic equilibrium theory (Clairaut's theory).

In contrast ERA-2005 rejects the "classical" Earth models. The ERA theory is such that even the non-rotating Earth has to have ellipticity of about $(2/3)e$, as the centrifugal potential of the steadily rotating Earth can account for only $\approx (1/3)e$ with k taken as k_2 (instead of k_s) even under constant forcing. No mechanism is proposed to determine the detailed interior structure.

There are large deviations of the ERA estimates for some of the Earth parameters from values computed on the basis of seismological Earth models (e.g. the ERA estimate for e_f being higher than e instead of being significantly lower).

The ERA model postulates a fluid inner core region which has the same physical properties as the fluid outer core, and yet is considered dynamically distinct from the latter.

As a model for the dynamics of the Earth's interior is essential for computing the non-rigid Earth nutations, the use of a non-realistic Earth model prevents us from computing correct nutations.

3.4. Estimation of Parameters

In the MHB 2000 solution, seven parameters were treated as adjustable and employed for fitting the theoretical outputs to the VLBI. The chosen parameters were the four Earth parameters e, e_f, k, γ, and three of the four real and imaginary parts of the the complex coupling constants K^{CMB} and K^{ICB} coefficients. The nutation amplitudes are influenced independently by these parameters, and are expected to be sensitive enough to permit estimation of their values. Minor empirical adjustment of the input values of one of the two parameters in the anelasticity model and one parameter in the ocean tide model was done at the end; and the real and imaginary parts of the prograde annual amplitude were taken as adjustable; the obliquity rate was also fitted to VLBI.

In the ERA-2005 solution, the estimated parameters are:

(i) e and k_2, (ii) e_c, k_2^c, k_2^v and α, which have high mutual correlations, (iii) ν_v, k_{el} and k_{dis} which do not appear in any of the resonance frequencies, (iv) four other "geophysical" parameters together with six "empirical" parameters, such as E_1 in order to adjust the ratio of the retrograde and prograde 18.6 y nutation amplitudes and E_2 to adjust the out-of-phase nutations.

The FCN frequency as obtained by earlier authors from their own fits is used as an input constraint while estimating the ERA parameters.

It should be noted that a low RMS of residuals may be produced by using a large number of adjustable parameters for estimation, but the presence of high correlations among, and lack of clear physical meaning of, many parameters makes the solution unreliable.

4. Comparisons of the Models with VLBI Observations

Residuals have been computed by subtracting time-series of VLBI observed celestial offsets from IAU precession-nutation predictions.

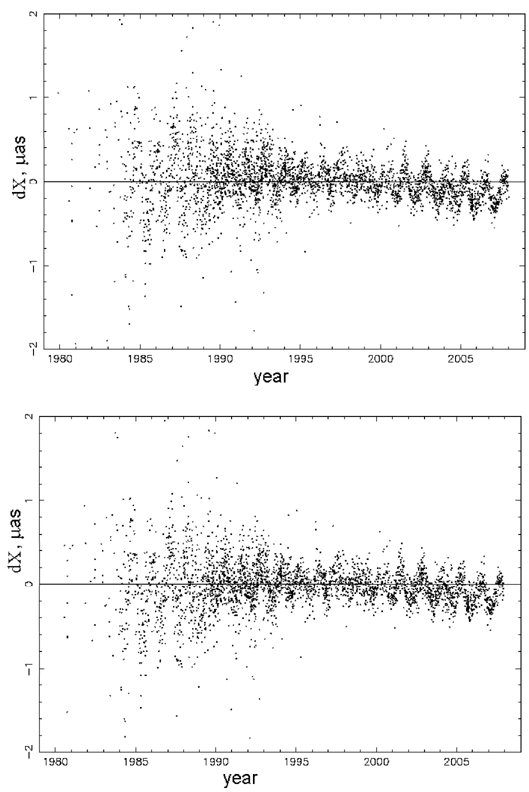

Figure 1. dX residuals of the IAU 2000 precession-nutation (top) and P03 precession plus IAU 2000A nutation (bottom) w.r.t. the IAA VLBI time series

We used either the IAU 2000A precession-nutation, or the P03 precession plus the (P03-adjusted) IAU 2000A nutation. Several data centers have been used as well as combined IVS series based on different procedures. Different empirical models were used for removing the free core nutation, one of them being a nutation of time-varying amplitude but constant period of -430.23 days. In all cases, after removal of the FCN, a "curvature" is visible in the two CIP coordinates X and Y in the GCRS, with approximately the same amplitude and the same phase (modulo π); the mean RMS is of the order of 125 μas in X and 170 μas in Y.

Fig. 1 provides two plots for residuals from the IAA time series (IAU 2000 and P03) that are almost indistinguishable by eye. Fig. 2 provides the VLBI residuals (with respect to GSFC time series) for the X and Y components, respectively, after the free core nutation has been removed.

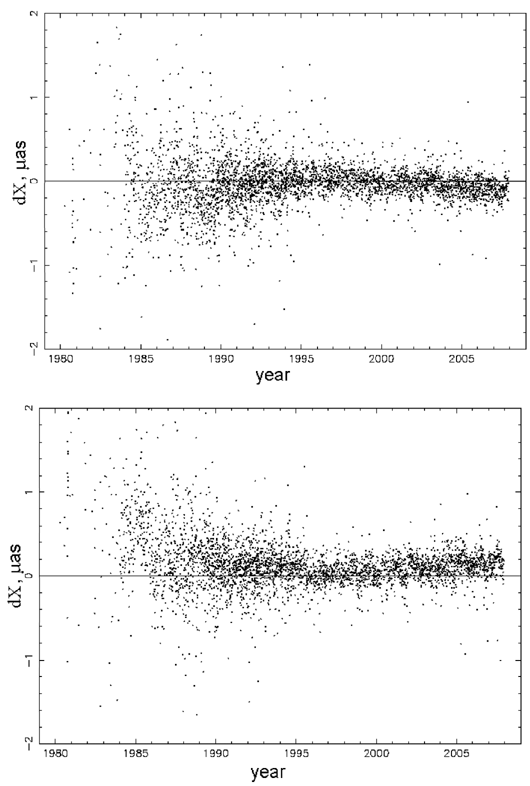

Figure 2. The residuals dX (top) and dY (bottom) of the IAU 2000/2006 precession-nutation w.r.t. GSFC VLBI time series after removing the FCN

We have looked at different possibilities to explain the observed form of the residuals. The effects due to celestial frame instability or due to the inaccuracy in the quadratic terms of the model were shown to be much too small.

One way to reduce the "curvature" was to remove a parabola fitted to the residuals. Tabl. 1, which provides results for different data centres, models and IVS combined series, shows that the coefficients of the fitted parabola are very sensitive to the VLBI time-series. It should be noted that interpreting the residuals in terms of a parabola was only for simplification and that the fit of a linear term plus a correction to a 18.6 y periodic term with an amplitude of a few tens of microarcseconds was shown to be as efficient as a parabola. In both cases, the post-fit RMS are reduced by about 10% in X and 30% in Y w.r.t. the pre-fit RMS. More reliable interpretation will only be possible with a longer interval of observations.

A recent study [17] also indicates that the curvature may be partly absorbed by a combination of long-period nutations with very small changes in their amplitudes and/or phases.

Table 1. Weighted fits of a parabola in the residuals corresponding to the IAU 2000 model and the P03/IAU 2000 models for different data centres (from 1980 to 2008) or various IVS combined series; unit: μas

		t^0	t	t^2
IAU2000−IAA	dX	-4 ± 3	-1172 ± 43	-6533 ± 787
IAU2000−OPA	dX	-43 ± 2	-862 ± 35	-2686 ± 679
P03−IAA	dX	-2 ± 3	-1004 ± 43	-9465 ± 787
P03−GSFC	dX	3 ± 3	-579 ± 38	-11217 ± 671
IAU2000−IAA	dY	26 ± 3	1287 ± 45	17428 ± 799
IAU2000−OPA	dY	16 ± 2	1422 ± 35	18984 ± 663
P03−IAA	dY	26 ± 3	774 ± 43	17401 ± 799
P03−GSFC	dY	58 ± 3	448 ± 40	14940 ± 690
IAU2000−`ivs07q4e`	converted to dX	-12 ± 3	-1113 ± 43	-11181 ± 718
IAU2000−`ivs07q4e`(*)	dX	-11 ± 3	-1261 ± 60	-13113 ± 873
IAU2000−`ivsM6q4e`(*)(**)	dX	-19 ± 3	-1086 ± 77	-8944 ± 893
IAU2000−`ivs07q4e`	converted to dY	51 ± 3	1529 ± 44	15970 ± 708
IAU2000−`ivs07q4e`(*)	dY	51 ± 3	1548 ± 60	16169 ± 873
IAU2000−`ivsM6q4e`(*)(**)	dY	33 ± 3	1639 ± 74	21729 ± 824

(*) truncated at 24/Oct/05; (**) originally referred to IAU 1976/1980 precession-nutation.

5. Comparisons Between the P03 Precession and INPOP06

The numerical planetary ephemeris INPOP06 [16] provides a solution for the precession-nutation of the axis of angular momentum, which takes into account the external torque acting on a non-rigid Earth model. A comparison over 5000 years of the P03 precession with INPOP06 showed a good agreement up to 1000 years. The large discrepancies appearing after that limit are due to the polynomial representation of the P03 semi-analytical precession.

A comparison over 400 years of the P03 precession/IAU 2000A nutation with the INPOP06 numerical precession-nutation shows no quadratic differences. This can be considered as a valuable check of the P03 quadratic terms as the two solutions are totally different and are based on totally independent models (except for the linear term in X which was fitted to P03).

6. Comparison Plots in the Time Domain

The effectiveness of the ERA-2005 theory as a predictive tool can be tested by comparing the projections made in 2006 [3, 18] with the subsequent VLBI observations.

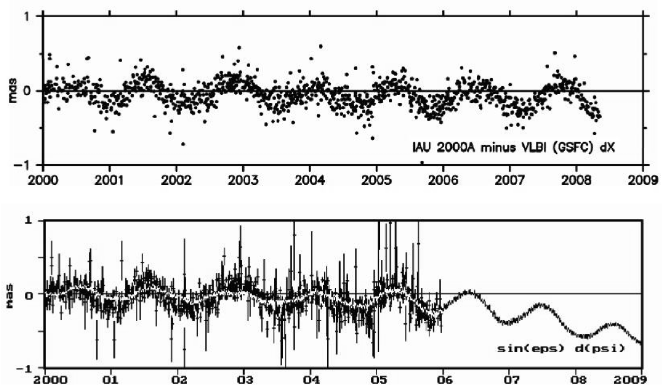

Figure 3. Comparison between the models and VLBI. Top: differences between the IAU 2000 predictions (which do not include FCN) and VLBI observations. Bottom (from [18]): differences between the IAU 2000 predictions and VLBI observations (points and error bars) and between IAU 2000 and ERA-2005 (line)

Fig. 3 provides comparison plots of the IAU 2000A and ERA-2005 models with the VLBI GSFC time series. The solid line in the lower plot (reproduced from [18]) represents the $d\Delta\psi \sin\epsilon$ differences of IAU 2000 with respect to ERA-2005 for the period 2000-2009. The runaway increase in these predicted differences beyond 2006 were attributed by this author to increasing errors in the IAU 2000 predictions. However, the actual differences dX $(= d\Delta\psi \sin\epsilon)$ of IAU 2000 with respect to VLBI observations, represented by the dots in the upper plot, do not show any significant increase in the deviation between IAU 2000 and VLBI measurements. So the large slope appearing in the solid line after 2006 in the lower plot is a measure of the failure of the predictions from the ERA-2005 theory. Moreover, we note that although the ERA-2005 model for the free core nutation model matches throughout the observed period (i.e. up to the end of 2006), though with insufficient amplitude, the extrapolations to the present go badly wrong. In the opinion of the authors, this comparison casts doubt on the physical reality of ERA-2005 and, by implication, subsequent models constructed in the same manner.

7. Summary

In this study devoted to comparisons of precession-nutation models:

- we have reviewed the key characteristics of the IAU 2000/2006 precession-nutation model and shown that no significant discrepancies appear between the IAU models and other semi-analytical solutions. We have also reported on the very good agreement between the P03 precession and the completely independent INPOP06 numerical integration [16];

- comparison between the IAU and ERA-2005 models has confirmed the severe deficiencies in the ERA-2005 theory [2] and its fit to data [3], which were pointed out in a recent comments paper [1];
- comparison of the IAU models with VLBI observations has shown an unexplained "curvature". As quadratic discrepancies do not seem physically possible, we can suspect an unmodeled long-periodic effect, or a spurious effect of the analyses; this shows that the IERS celestial offsets are still necessary to predict precession-nutation with microarcsecond accuracy.

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